EXPERIMENTAL AND SIMULATIONAL APPROACH TO DISCRETE AND CONTINUUM PERCOLATION PROBLEMS

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A fact that discrete and continuum percolation models belong to the different universality class is proved by experiments and simulation which are related to the electric conductivity, the permeability, the elasticity and the normal vibrational mode energy.

§ 1. Introduction

In 1957, Broadbent and Hammerseley introduced so called percolation problem on the discrete lattice, for the first time.[1] This is a very simple and basic model which shows a prominent change in various physical quantities as well as Ising spin model. Most of physicists thought that a continuum percolation system can be approximated by introducing an infinitely fine lattice. However, some differences are noticed by observing materials in nature, most of which can be described by continuum percolation problems. Halperin et al. pointed out that critical exponents of the percolation problems might be different between the discrete and the continuum percolation models.[2] The main conclusion is listed in Table 1 for the electric conductivity, the elastic constant and the permeability in the two- and three-dimensions.

We have some variations of the original percolation problem. In Fig.1a and 1b, two kinds of continuum percolation models are shown. The model shown in Fig.1a is called Swiss-cheese model because of the structure which is similar to Swiss-cheese and the other one shown in Fig.1b is inverted Swiss-cheese model. The exponents of the electric conductivity for discrete, continuum (Swiss-cheese) and inverted Swiss-cheese models are the same in the two-dimension. However, the exponents for the three-dimensional models are different each other.

Table 1. The critical exponents for electric conductivity, elastic constant and permeability of the discrete, the continuum Swiss-cheese and the continuum inverted Swiss-cheese percolation models in the two- and three-dimensions.[2]

model		electric conductivity	elastic constant	permeability
two dimension	discrete	1.3	3.7	1.3
	Swiss	1.3	5.2	2.8
	inverted Sw.	1.3	3.7	1.3
three dimension	discrete	1.9	3.6	1.9
	Swiss	2.4	6.1	4.4
	inverted Sw.	1.9	4.1	2.4

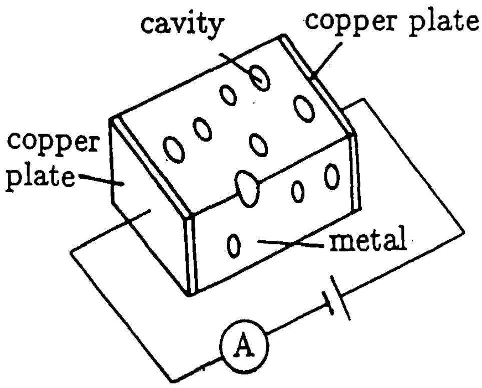

Fig. 1a Swiss-cheese Model

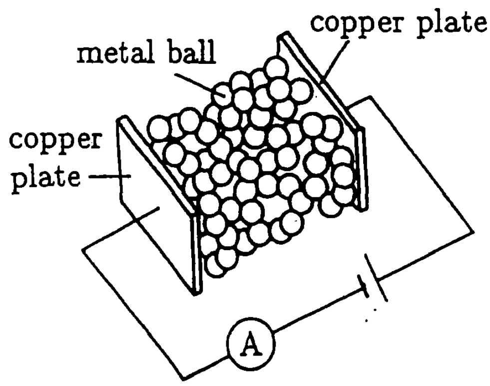

Fig. 1b Inverted Swiss-cheese Model

Although it is easy to punch off holes in the two-dimensional system such as the aluminum sheet, it is difficult to make cavities inside the three-dimensional material and take it off to outside without leaving any damage inside the object. In 1991, we proposed a three-dimensional experiment of Swiss-cheese percolation with a simple idea, which gets rid of the above mentioned difficulty. At first, we set electrically conductive fluid such as mercury or pure water in a container. If we push rubber balls into the fluid, the electrically conductive fluid spills over from the container by the same volume as the rubber balls. The electric conductivity decreases as the number of balls which are pushed into the fluid increases. The conductivity is reduced to zero at a threshold volume ratio of fluid component.

In the next section, we discuss the exponent of electrical conductivity and permeability of three-dimensional Swiss-cheese model, and we can show the exponents are 2.4 and 4.2, respectively, which are equivalent values to the theoretical prediction. In § 3, we discuss an inverted Swiss-cheese model in which the elastic constant is an object of investigations. Furthermore, we discuss an simulational approach to the Swiss-cheese model for the first time.

§ 2. Exponent of Swiss-cheese percolation model

The electric conductivity of lime stone was measured and gave a clue to understanding of differences between the discrete and continuum percolation models. In 1991, we showed an experimental evidence which revealed the difference between the discrete and the continuum percolation models, avoiding the above mentioned difficulty by introducing electrically conductive fluid instead of solid material.[3] Although it is very difficult to make cavities in the solid material and to take it off from the material, we can simulate the situation by inserting rubber balls which play role of cavities into fluid.

Our apparatus is shown in Fig.2, where a container is an acrylic tube of diameter 10cm and contains 360 rubber balls (the diameter is 16mm). A piston at the right hand side is used to control the space among the rubber balls filled by the electrically conductive fluid, i.e. if one presses the piston, the gap becomes narrow and therefore the electric conductivity of fluid between both the ends of acrylic tube decreases. Only one trial gave the exponent of electric conductivity 2.4 ± 0.1 together with very good reproducibility.

By using similar apparatus shown in Fig.3 we can measure the permeability of fluid through the porous object made by balls pressed by the piston. In this case we obtained the exponent of fluid permeability 4.2 ± 0.3 [4], which should be compared with the predicted value 4.4 shown in Table 1.

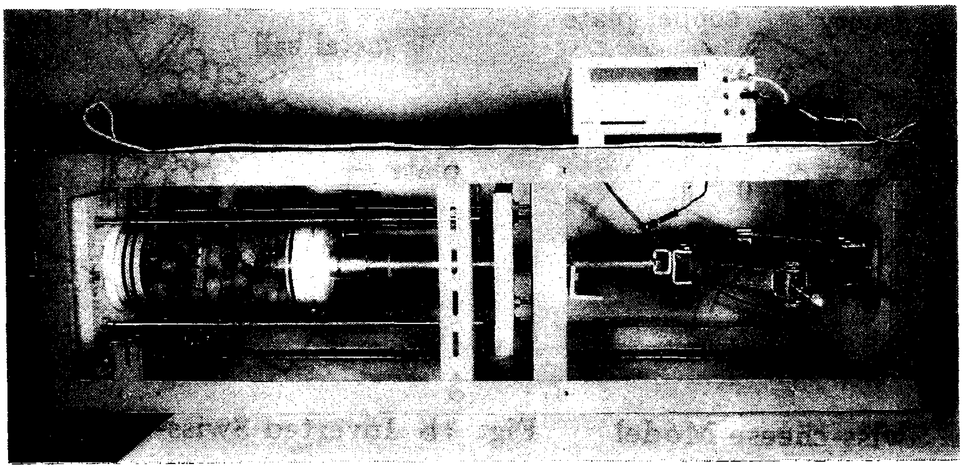

Fig.2. Apparatus for electric conductivity in the three-dimensional Swiss-cheese model.

§ 3. Exponent of inverted Swiss-cheese percolation model[5)]

Here we consider an elastic system which consists of many rubber balls. When the rubber balls form a zigzag chain from one side to aother of a container by adding one by one (see Fig.4), then the chain shows an elastic property against pressing the piston at the one side of the container. One of the difficulties for realization of this inverted Swiss-cheese model is that we have to lift up balls in the container without any support. Here we introduce a randomly netted pile of very thin copper wire (so called enamel wire). Many balls are hung up with the wire, which has a negligibly small elastic constant. This is shown in Fig.4.

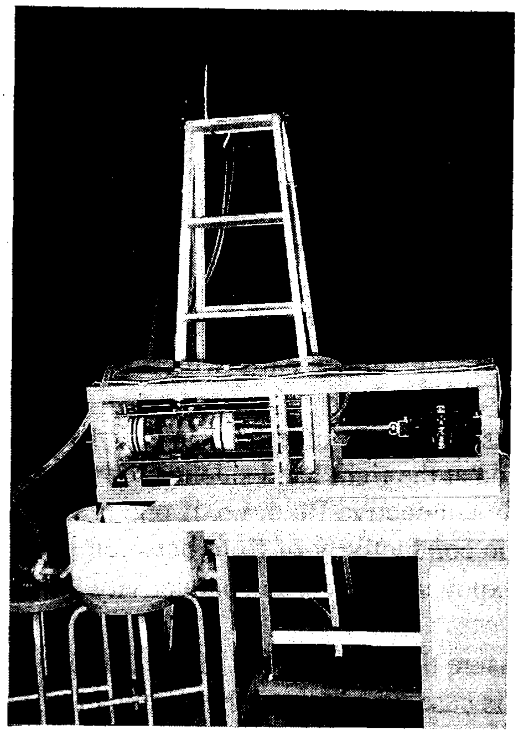

Fig.3. Apparatus for permeability in the three-dimensional Swiss-cheese model.

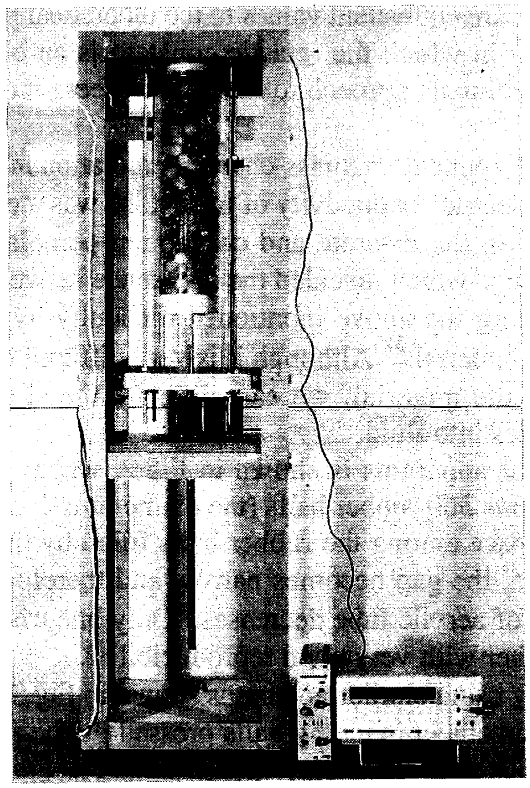

Fig.4. Apparatus for elasticity in the three-dimensional inverted Swiss-cheese model.

At first there is no sequence of balls from a side to another of the container. When we press the one side by the piston into the container, the rubber balls get contact each other gradually and form a sequence of rubber balls or zigzag chain. As soon as the sequence is formed, if we press the side a little, stress can be measured at the other side with load transducer. Calculating the elastic constant from the data and plotting them by a log-log plot, we obtain the exponent 4.15, which is comparable with the predicted exponent 4.1 in Table 1.

The two-dimensional Swiss-cheese model is simulated by the computer. Here we discuss the percolation probability and the connectivity. We obtain the threshold value $p_c = 0.315$ and $\nu = 1.65$ for the first time for the Swiss-cheese model. The detail will be given in a separate paper.

§ 4. Summary and discussion

Up to here we consider the electric conductivity and permeability in Swiss-cheese model and the elasticity in inverted Swiss-cheese model. It is clearly shown in these experiments that there are differences between the discrete and the continuum percolation models. However, if one looks at Table 1, everyone easily notices that there are still many exponents which are not experimentally checked.

Furthermore there are still more physical quantities in solids such as vibration, specific heat, and so on. These quantities also change when the porosity of solids changes. For example, the lowest energy of a normal vibration modes of aluminum plate is measured as the porosity is changed. We an already noticed decrease of vibrational energy with porosity and it is also clear that we have a threshold value that the mode energy becomes zero. In a similar way, every physical quantities depend on the porosity and there should exist the threshold at somewhere of the porosity.

Coupled with theoretical and experimental investigations, nowadays simulation is also another very important method for research. Simulation study of the continuum percolation model is not difficult in principle, but very difficult in practice even in two-dimensional space. Here we discuss the connectivity of random circle holes of various radius. By using finite size scaling technique, the threshold value and the exponent of correlation length are obtained as $p_c = 0.315$ and $\nu = 1.65$, respectively. The exponent is different from 4/3 for the discrete percolation model[6] Thus we have been able to show the difference between the discrete and the continuum percolation models in its critical exponents of the electric conductivity, the permeability and the elasticity. In the discrete and the continuum percolation model, there are many interesting and stimulus points of study and there remain a lot of subjects which are not clear.

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